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# INVERSE IDENTIFICATION OF DAM-RESERVOIR INTERACTION INCLUDING THE EFFECT OF RESERVOIR BOTTOM ABSORPTION

J. Nasserzare[*1], Y. Lei[2], and F. Ziegler[1]
[1]Department of Civil Engineering, Technical University Vienna / E201,
A- 1040 Vienna, Austria
[2]Center For Embedded Networked Sensing, University of California, Los Angeles, CA, USA

## ABSTRACT

In this paper, a procedure is developed which can be used to identify the natural frequencies and natural modes in vacuum of an Arch-Dam from forced vibration testing data of partially filled reservoir. The effect of hydrodynamic pressure is removed by using an efficient algorithm. To verify the procedure, a simple structure is substituted for the dam with known properties in vacuum. Then a thin SSSF-plate is considered as the retaining wall representing of the dam and a sub-structuring technique is used with regard to a three dimensional linear compressible inviscid fluid body. The calculated resonance in the illustrated example replaces the resonance which in practical in-situ has been measured. Also the effect of the wave absorption at the bottom and bank of the reservoir is considered. The hydrodynamic pressure of the reservoir is calculated using boundary element method. The results which derived by solving an inverse problem, are compared with the exact analytical responses of the plate.

**Keywords:** dam-reservoirs, absorption, arch dams, plate, boundary element method

## 1. INTRODUCTION

One of the main topics in earthquake engineering is the dynamic analysis of Dam-Reservoir systems. There are several extensive methods, which have been applied during the last decades. Numerical procedures which include the interaction between several domains with different properties: concrete dam, foundation rock, water, bottom sediments and bank of the reservoir, have been developed by using the finite element method, the boundary element method and various combinations of both methods [1,4,11,2]. Due to the difficulty in determining the dynamic model of foundation rock, and especially the high sensitivity of the dynamic response to the selection of the dynamic Young's modules of dam concrete structure, using a linear model is the basis for any preliminary dynamic analysis of dam-reservoir systems.

In addition to the theoretical studies, there have been several in-situ experimental studies performed on dams [7,8]. The forced vibration tests can yield reliable data from which identification of suitable models of the dynamic behavior of the concrete dam, and the dam-foundation interaction effectively can be made to control and adjust the assumptions made in

[*] E-mail address of the corresponding author: jn@allmech.tuwien.com

numerical calculations. As a contribution to the UN-IDNDR, the large-scale dynamic models of the interactive system "dam-water-soil" are investigated within the project "Safety of Dams in a Seismically Activated Environment" (organized by F. Ziegler, Technical University of Vienna), to further improve the safety standards. The project was initiated during the summer quarter 1992 by performing vibration tests of the large arch dam "Schlegeissperre" (erected 1967–1971), situated in Zillertal, Tyrol, Austria in the presence of the authors.

Three series of vibration tests have been performed during the year 1993-95, at different water levels. Data analysis was carried out by means of FFT and the properly modified program MODAL 3.0SE (Make SMS) to obtain resonance frequencies and resonance modes of the dam-reservoir-system [9, 10].

Since in most cases it is not practicable to perform dynamic in-situ tests with an empty reservoir, it would be necessary to remove the effect of the hydrodynamic pressure on the dam. As part of the research work within the project, this investigation is performed by solving an inverse problem using the data obtained from the forced vibration tests.

In a first step and for verifying the solution method, the dynamic response of simple structures like beams and plates with fluid interaction effect taken into account are analyzed, since there are analytical results of the modal properties of the structure available for comparison purpose [12].

An efficient and accurate solution method for interactive vibration problems of simple structures with fluid by [5] has been developed. In this paper the method is extended to the radiation damping due to the out going waves. The damping effect is introduced by absorption of energy at flexible foundation and banks of the reservoir.

## 2. TIME-HARMONIC VIBRATIONS OF THE FLUID BODY IN AN IRREGULARLY SHAPED RESERVOIR

*Assumptions* - The Arch dam-reservoir-foundation system is replaced by a SSSF-plate facing a water domain with the following assumptions:

- Small amplitude motion,
- Water is compressible and in viscid,
- Effect of surface waves are neglected,
- Waves travel identical throughout the bottom and banks in the

*Equations of Motion* **-** For a Time-harmonic excitation and by considering the above assumptions, the hydrodynamic pressure is a result of the three-dimensional Helmholtz equation:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)p+k^2 p=0 \tag{1}$$

where,

$x, y, z$ = Cartesian coordinates

$k$ = wave number

$= \dfrac{\nu}{C_f}$

$\nu$ = exciting frequency
$C_f$ = sound velocity in fluid.( for water = 1484 m/s )
p = hydrodynamic pressure

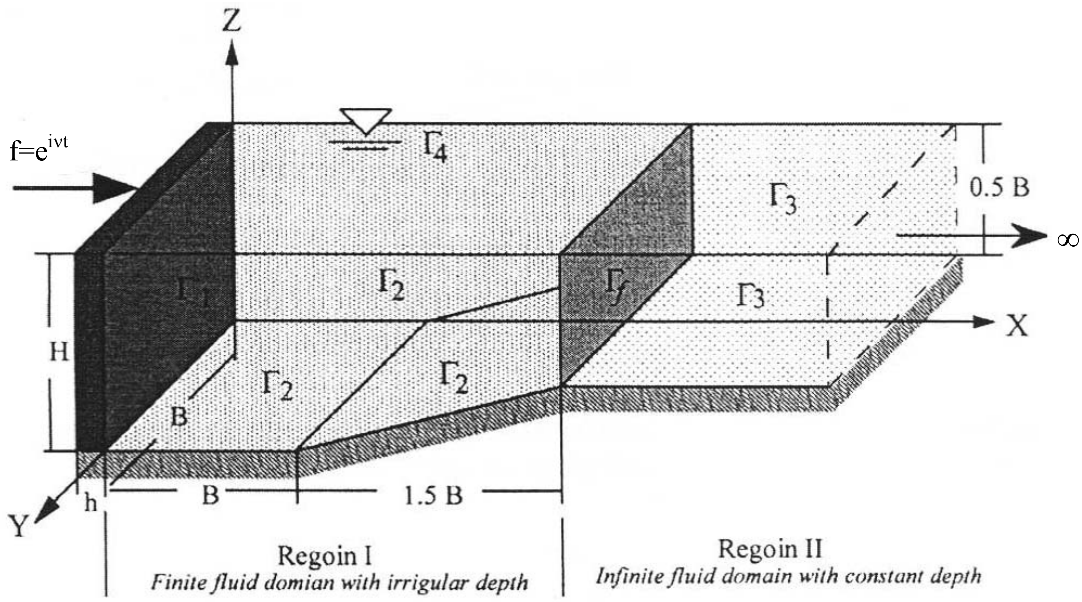


Figure 1. Dam-Reservoir mathematical model

*Boundary Conditions*-The boundary conditions associated with Eq.1 can be expressed as follows:

at the dam-fluid interface $\Gamma_1$

$$\frac{\partial p_1}{\partial n} = \rho_f \nu^2 w_n \tag{2a}$$

where,

$p_1$ = hydrodynamic pressure at the dam-fluid interface
n = outward normal to the interface
$\rho_f$ = density of fluid
$w_n$ = normal component of the dam displacement at the boundary

at the reservoir bottom and banks $\Gamma_2$

$$\frac{\partial p_2}{\partial n} = \rho_f \nu^2 w_n + i\nu\gamma p_2 \tag{2b}$$

where,

$p_1$ = pressure at the fluid-foundation boundary

$\gamma$ = damping coefficient

$= \frac{1}{C_f}\left(\frac{1-\alpha_R}{1+\alpha_R}\right); 0 \le \gamma \le$

$\alpha_R$ = wave reflection coefficient ; $0 \le \alpha_R \le 1$

at the surface of the reservoir $\Gamma_3$

$$p_3 = 0 \tag{2c}$$

along the Near-Far field interface $\Gamma_f$

It is located at the interface of irregular finite section, near field (Region $I$ ) and an uniform infinite section, far field (Region $II$ ) . There exist the following equilibrium condition,

$$\begin{cases} p_f^I = p_f^{II} \\ \dfrac{\partial p_f^I}{\partial n} = -\dfrac{\partial p_f^{II}}{\partial n} \end{cases} \tag{2d}$$

where, $p_f$ is the hydrodynamic pressure at $\Gamma_f$.

For a efficient numerical solution of the problem, governed by Eq.(1), the third Green's identity is used to transform the three-dimensional mixed boundary value problem into a two-dimensional boundary integral equation:

$$\oint_{\Gamma}\left[G(\vec{r}-\vec{r}_0)\frac{\partial p(\vec{r}_0)}{\partial n_0} - p(\vec{r}_0)\frac{\partial G(\vec{r}-\vec{r}_0)}{\partial n_0}\right]d\Gamma = \begin{cases} p(\vec{r}) \ ; \ \vec{r} \in V \\ 0 \quad ; \ \vec{r} \notin V \end{cases} \tag{3}$$

where the Green's function has to satisfy the inhomogeneous Helmholtz equation, Eq.(4), with the Dirac delta function on the right hand side

$$\nabla^2 G(\vec{r}-\vec{r}_0) + k^2 G(\vec{r}-\vec{r}_0) = -\delta(\vec{r}-\vec{r}_0) \tag{4a}$$

$$\iiint_V \delta(\vec{r}-\vec{r}_0)dV = \begin{cases} 1 ; \vec{r} \in V \\ 0 ; \vec{r} \notin V \end{cases} \tag{4b}$$

$\vec{r}$ = the position vector of the observation point

$\vec{r}_0$ = the position vector of the source point

$\Gamma$ = the closed surface bounding the interior volume $V$

$\dfrac{\partial}{\partial n_0}$ = differentiation along the outward normal to $\Gamma$

Höllinger [3] has applied the following fundamental solution, which satisfies implicitly the free surface condition at $\Gamma_3$:

$$G(\vec{r}, \vec{r}_0) = \frac{1}{4\pi}\left(\frac{e^{ik\rho}}{\rho} - \frac{e^{ik\rho'}}{\rho'}\right) \tag{5a}$$

where

$$\rho = |\vec{r} - \vec{r}_0| = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \tag{5b}$$

$$\rho' = |\vec{r} - \vec{r}_0'| = \sqrt{\rho^2 + 4zz_0} \tag{5c}$$

The boundary $\Gamma$ is discretized and then approximated by dividing it into $m$ plane elements $A_j (1 \le j \le m)$. The hydrodynamic pressure in the central point of each element is sampled in a vector $p$, thus obtaining a vector with $m$ unknown components $p_j$. Therefore, applying Eq. (3) renders a set of linear equations of hydrodynamic pressure vector $\vec{p}$ :

$$\sum_{j=1}^{m} A_{ij} p_j = \sum_{j=1}^{m} B_{ij} \frac{\partial p(\vec{r}_j)}{\partial n_j} \tag{6}$$

where,

$$A_{ij} = \iint_{\Gamma_j} \frac{\partial G(\vec{r}_i - \vec{r}_{0j})}{\partial n_{0j}} d\Gamma_j \tag{7a}$$

$$B_{ij} = \iint_{\Gamma_j} G(\vec{r}_i - \vec{r}_{0j}) dA_j \tag{7b}$$

Thus the whole set of equations in the Region *I*, enclosed by $\Gamma_1$, $\Gamma_2$ and $\Gamma_f$, can be expressed in a matrix form as:

$$\begin{bmatrix} \mathbf{A}^{\mathrm{I}}_{11} & \mathbf{A}^{\mathrm{I}}_{12} & \mathbf{A}^{\mathrm{I}}_{1\mathrm{f}} \\ \mathbf{A}^{\mathrm{I}}_{21} & \mathbf{A}^{\mathrm{I}}_{22} & \mathbf{A}^{\mathrm{I}}_{2\mathrm{f}} \\ \mathbf{A}^{\mathrm{I}}_{\mathrm{f}1} & \mathbf{A}^{\mathrm{I}}_{\mathrm{f}2} & \mathbf{A}^{\mathrm{I}}_{\mathrm{ff}} \end{bmatrix} \begin{Bmatrix} \mathbf{p}^{\mathrm{I}}_{1} \\ \mathbf{p}^{\mathrm{I}}_{2} \\ \mathbf{p}^{\mathrm{I}}_{\mathrm{f}} \end{Bmatrix} = \begin{bmatrix} \mathbf{B}^{\mathrm{I}}_{11} & \mathbf{B}^{\mathrm{I}}_{12} & \mathbf{B}^{\mathrm{I}}_{1\mathrm{f}} \\ \mathbf{B}^{\mathrm{I}}_{21} & \mathbf{B}^{\mathrm{I}}_{22} & \mathbf{B}^{\mathrm{I}}_{2\mathrm{f}} \\ \mathbf{B}^{\mathrm{I}}_{\mathrm{f}1} & \mathbf{B}^{\mathrm{I}}_{\mathrm{f}2} & \mathbf{B}^{\mathrm{I}}_{\mathrm{ff}} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \mathbf{p}^{\mathrm{I}}_{1}}{\partial \mathrm{n}} \\ \dfrac{\partial \mathbf{p}^{\mathrm{I}}_{2}}{\partial \mathrm{n}} \\ \dfrac{\partial \mathbf{p}^{\mathrm{I}}_{\mathrm{f}}}{\partial \mathrm{n}} \end{Bmatrix} \tag{8}$$

The relation between p and $\dfrac{\partial p}{\partial n}$ along $\Gamma_{\mathrm{f}}$ of Region *II* is :

$$\mathrm{p}(\mathrm{x},\mathrm{y},\mathrm{z})\big|_{\Gamma_{\mathrm{F}}} = \mathrm{I}_{\mathrm{nj}}\, \mathrm{e}^{\left(-\mathrm{x}\sqrt{\lambda_{\mathrm{j}}^{2}-\mathrm{k}^{2}}\right)} \cos(\lambda_{\mathrm{j}}\mathrm{z}) \mathrm{A}_{\mathrm{j}}(\nu) \tag{9}$$

where,

$$\mathrm{I}_{\mathrm{nj}} = \int_{\mathrm{O}}^{\mathrm{H}} \phi_{\mathrm{j}}(\mathrm{z}) \cos(\lambda_{\mathrm{j}}\mathrm{z})\mathrm{dz} \tag{10}$$

and $\mathrm{A}_{\mathrm{j}}(\nu)$ is an unknown coefficient which eliminates after applying boundary condition in Eq. (2d).

Note that Eq. (9) satisfies the equation of motion and the boundary conditions along $\Gamma_2$ and the free surface, $\Gamma_3$. By applying Eqs. (6) and (9) we have the relation between $p$ and $\dfrac{\partial \mathrm{p}}{\partial \mathrm{n}}$ along $\Gamma_{\mathrm{f}}$ of Region *I* written in a matrix form as follows:

$$\begin{Bmatrix} \mathbf{p}^{\mathrm{I}}_{1} \\ \mathbf{p}^{\mathrm{I}}_{2} \\ \mathbf{p}^{\mathrm{I}}_{\mathrm{f}} \end{Bmatrix} = \begin{bmatrix} \mathbf{D}^{\mathrm{I}}_{11} & \mathbf{D}^{\mathrm{I}}_{12} & \mathbf{D}^{\mathrm{I}}_{1\mathrm{f}} \\ \mathbf{D}^{\mathrm{I}}_{21} & \mathbf{D}^{\mathrm{I}}_{22} & \mathbf{D}^{\mathrm{I}}_{2\mathrm{f}} \\ \mathbf{D}^{\mathrm{I}}_{\mathrm{f}1} & \mathbf{D}^{\mathrm{I}}_{\mathrm{f}2} & \mathbf{D}^{\mathrm{I}}_{\mathrm{ff}} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \mathbf{p}^{\mathrm{I}}_{1}}{\partial \mathrm{n}} \\ \dfrac{\partial \mathbf{p}^{\mathrm{I}}_{2}}{\partial \mathrm{n}} \\ \dfrac{\partial \mathbf{p}^{\mathrm{I}}_{\mathrm{f}}}{\partial \mathrm{n}} \end{Bmatrix} \tag{11}$$

where

$$\mathbf{D} = \mathbf{A}^{-1}\mathbf{B} \tag{12}$$

Hence,

$$\mathbf{p}^{\mathrm{I}}_{\mathrm{f}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{f}1}\left(\frac{\partial \mathbf{p}^{\mathrm{I}}_{1}}{\partial \mathrm{n}}\right) + \mathbf{D}^{\mathrm{I}}_{\mathrm{f}2}\left(\frac{\partial \mathbf{p}^{\mathrm{I}}_{2}}{\partial \mathrm{n}}\right) + \mathbf{D}^{\mathrm{I}}_{\mathrm{ff}}\left(\frac{\partial \mathbf{p}^{\mathrm{I}}_{\mathrm{f}}}{\partial \mathrm{n}}\right). \tag{13}$$

By applying the boundary condition in Eq. (2d), we obtain,

$$\left(\mathbf{D}_{\mathrm{ff}}^{\mathrm{I}}+\mathbf{D}_{\mathrm{ff}}^{\mathrm{II}}\right)\left(\frac{\partial \mathbf{p}_{\mathrm{f}}^{\mathrm{I}}}{\partial \mathrm{n}}\right)=\mathbf{D}_{\mathrm{f1}}^{\mathrm{I}}\left(\frac{\partial \mathbf{p}_{1}^{\mathrm{I}}}{\partial \mathrm{n}}\right)+\mathbf{D}_{\mathrm{f2}}^{\mathrm{I}}\left(\frac{\partial \mathbf{p}_{2}^{\mathrm{I}}}{\partial \mathrm{n}}\right) \tag{14}$$

substituting Eqs. (14) and (13) into Eq. (11), the equation for calculating hydrodynamic pressure on the up-stream face of the dam is:

$$\mathbf{p}_{1}^{\mathrm{I}}=\mathbf{D}_{11}^{\mathrm{I}}\left(\frac{\partial \mathbf{p}_{1}^{\mathrm{I}}}{\partial \mathrm{n}}\right)+\mathbf{D}_{12}^{\mathrm{I}}\left(\frac{\partial \mathbf{p}_{2}^{\mathrm{I}}}{\partial \mathrm{n}}\right)+\mathbf{D}_{1\mathrm{f}}^{\mathrm{I}}\left\{\left(\mathbf{D}_{\mathrm{ff}}^{\mathrm{I}}+\mathbf{D}_{\mathrm{ff}}^{\mathrm{II}}\right)^{-1}\left[\mathbf{D}_{\mathrm{f1}}^{\mathrm{I}}\left(\frac{\partial \mathbf{p}_{1}^{\mathrm{I}}}{\partial \mathrm{n}}\right)+\mathbf{D}_{\mathrm{f2}}^{\mathrm{I}}\left(\frac{\partial \mathbf{p}_{2}^{\mathrm{I}}}{\partial \mathrm{n}}\right)\right]\right\}. \tag{15}$$

## 3. REMOVING THE EFFECT OF THE HYDRODYNAMIC PRESSURE BY SOLVING AN INVERSE PROBLEM

In order to formulate the inverse problem, it is assumed that resonance frequencies and resonance modes of the dam-reservoir system are obtained from forced vibration tests and data analysis [10]. It is then required to remove the effect of hydrodynamic pressure to evaluate the natural frequencies and natural modes of the system.

The arch dam is assumed to be linear elastic. The continuous displacement field of the dam can be approximated by discrete displacements of a model with finite number of unstained nodal points. Thus, the equation of motion for the discrete displacements of a light damped dam under hydrodynamic pressure at the upstream face and under time harmonic external force excitation can be expressed in matrix form as:

$$\mathbf{M}\ddot{\mathbf{w}}(\mathrm{t})+\mathbf{C}\dot{\mathbf{w}}(\mathrm{t})+\mathbf{K}\mathbf{w}(\mathrm{t})=\mathbf{p}(\mathrm{t})+\mathbf{f}(\mathrm{t}) \tag{16}$$

where,

$\mathbf{M}$ = symmetric mass matrix
$\mathbf{C}$ = symmetric damping matrix
$\mathbf{K}$ = symmetric stiffness matrix
$\mathbf{p}(\mathrm{t})$ = vector of the nodal point loads associated with the hydrodynamic pressure
$\mathbf{f}(\mathrm{t})$ = vector of external excitation forces applied at some selected nodal points
$\ddot{\mathbf{w}}(\mathrm{t})$ = nodal point acceleration vector, relative to the ground
$\dot{\mathbf{w}}(\mathrm{t})$ = nodal point velocity vector, relative to the ground
$\mathbf{w}(\mathrm{t})$ = nodal point displacements vector, relative to the ground

where,

$$\mathbf{w}^{\mathrm{T}}=\{\mathrm{u}_1,\mathrm{v}_1,\mathrm{w}_1,\mathrm{u}_2,\mathrm{v}_2,\mathrm{w}_2,\ldots\mathrm{u}_n,\mathrm{v}_n,\mathrm{w}_n\} \tag{17}$$

in which,

u = component of $\mathbf{w}$ with respect to the x-axis
v = component of $\mathbf{w}$ with respect to the y-axis

w = component of $\mathbf{w}$ with respect to the z-axis

Under time-harmonic force excitation with frequency $\nu$, response is also time-harmonic with the same frequency $\nu$. Thus, Eq. (16) looks on the time reduced form:

$$-\nu^2\mathbf{M}\mathbf{w} + i\nu\mathbf{C}\mathbf{w} + \mathbf{K}\mathbf{w} = \mathbf{p} + \mathbf{f} \tag{18}$$

The deformation vector $\mathbf{w}$ can be approximated by the finite series of the (non-orthogonal) resonance modes of the coupled system:

$$\mathbf{w} = \sum_{j=1}^{n} \psi_j Y_j \tag{19}$$

where,

$\psi_j$ = the $j$th resonance mode of the coupled arch dam-fluid system

$Y_j$ = the $j$th associated coordinates

n = the number of modes selected in modal analysis.

The hydrodynamic pressure on the upstream face in the $j$th resonance mode of deformation $\mathbf{p}_j$ can be calculated by the boundary element method just by inserting the resonance mode $\psi_j$ and multiplying the resonance frequency $\tilde{\omega}_j$. The total hydrodynamic pressure of any forcing frequency can be expressed as the linear combination of $\mathbf{p}_j$:

$$\mathbf{P} = -\nu^2 \sum_{j=1}^{n} \mathbf{p}_j Y_j \quad ; \quad j = 1, 2, \ldots n \tag{20}$$

Therefore, Eq. (18) renders:

$$-\nu^2\mathbf{M}\mathbf{\Psi}\mathbf{Y} + i\nu\mathbf{C}\mathbf{\Psi}\mathbf{Y} + \mathbf{K}\mathbf{\Psi}\mathbf{Y} = \mathbf{P}\mathbf{Y} + \mathbf{f} \tag{21}$$

where,

$\mathbf{\Psi}$ = matrix of the resonance modes of the discretized arch dam-fluid

$\mathbf{P}$ = corresponding loading matrix associated with the hydrodynamic pressure

In resonance, the external excitation frequency $\nu$ equals the resonance frequency $\tilde{\omega}_j$, then Eq. (21) becomes:

$$-\tilde{\omega}_j^2\mathbf{M}\psi_j Y_j + i\tilde{\omega}_j\mathbf{C}\psi_j Y_j + \mathbf{K}\psi_j Y_j = \mathbf{p}_j Y_j + \mathbf{f}_j \quad ; \quad j = 1,\ 2,\ \ldots n \tag{22}$$

The resonance mode can be expanded into a series of the orthogonal set of natural modes in vacuum with unknown coefficients $\mathbf{d}_j$ as:

$$\psi_j = \mathbf{\Phi d}_j \quad ; \quad j = 1,\ 2,\ \ldots n \tag{23}$$

where,

$\mathbf{\Phi}$ = modal matrix of the undamped dam in vacuum

Substituting Eq. (23) into Eq. (22) renders:

$$-\tilde{\omega}_j^{\,2}\mathbf{M\Phi d}_j Y_j + i\tilde{\omega}_j \mathbf{C\Phi d}_j Y_j \ + \mathbf{K\Phi d}_j Y_j - \mathbf{p}_j Y_j = \mathbf{f}_j \tag{24}$$

Multiplying both sides of Eq. (24) by $\psi_i^{\mathrm{T}}$ which is also expanded into the natural modes of the dam analogous to Eq. (23), renders formally:

$$-\tilde{\omega}_j^{\,2}\mathbf{d}_i^{\,T}\mathbf{\Phi}^T\mathbf{M\Phi d}_j Y_j + i\tilde{\omega}_j \mathbf{d}_i^{\,T}\mathbf{\Phi}^T\mathbf{C\Phi d}_j Y_j \ + \mathbf{d}_i^{\,T}\mathbf{\Phi}^T\mathbf{K\Phi d}_j Y_j - \psi_i^{\,T}\mathbf{p}_j Y_j = \psi_i^{\,T}\mathbf{f}_j \tag{25}$$

Using the orthogonality of the natural modes Eq. (25) renders to the set of equations:

$$\left[\left\{\sum_{k=1}^{n}\left(\omega_k^{\,2} - \tilde{\omega}_j^{\,2} + 2i\xi_k\omega_k\tilde{\omega}_j\right) d_{ki}\, d_{jk}\right\} - \psi_i^{\,T}\mathbf{p}_j\right] Y_j = \psi_i^{\,T}\mathbf{f}_j$$
$$i = 1,\ 2,\ \ldots n \quad ; \quad j = 1,\ 2,\ \ldots n \quad ; \quad \nu = \tilde{\omega}_j \quad ; \quad i = \sqrt{-1} \tag{26}$$

where the natural modes of the arch dam are assumed orthonormalized

$$\mathbf{\Phi}^T\mathbf{M\Phi} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} = \mathbf{I} \tag{27}$$

where, $\mathbf{I}$ = unit matrix

and the damping matrix $\mathbf{C}$ is assumed to satisfy the matrix condition:

$$\mathbf{\Phi}^T\mathbf{C\Phi} = \begin{bmatrix} 2\xi_1\omega_1 & 0 & \cdots & 0 \\ 0 & 2\xi_2\omega_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 2\xi_n\omega_n \end{bmatrix} \quad ; \quad \xi_1 \le \xi_2 \le \cdots \le \xi_n \tag{28}$$

where, $\xi$ = modal damping ratio.

Excitation is a single concentrated force at the crown of the dam:

$$f_{ij}(y,t) = \psi_i^T F_j = f_{ij}\delta(y - a_0)e^{i\tilde{\omega}_j t} \tag{29}$$

and

$$\xi\, p_{ij} = \psi_i^T p_j \tag{30}$$

Then Eq. (26) reduces to:

$$\left[\left\{\sum_{k=1}^{n}\left(\omega_k^{\,2} - \tilde{\omega}_j^{\,2} + 2i\,\xi_k\,\omega_k\,\tilde{\omega}_j\right)d_{ki}\,d_{jk}\right\} - \tilde{\omega}_j^{\,2} p_{ji}\right]Y_j = \psi_i(a_0)\, f_{ij} \tag{31}$$

Multiplying both sides of Eq.(31) by $\left\{\dfrac{\psi_j(a_0)}{\psi_j(a_0)\cdot\psi_i(a_0)}\right\}$ and using Eq.(19), renders formally:

$$\frac{1}{\psi_j(a_0)\cdot\psi_i(a_0)}\left[\left\{\sum_{k=1}^{n}\left(\omega_k^{\,2} - \tilde{\omega}_j^{\,2} + 2i\,\xi_k\,\omega_k\,\tilde{\omega}_j\right)d_{ki}\,d_{jk}\right\} - \tilde{\omega}_j^{\,2} p_{ji}\right]w_j(a_0) = f_{ij} \tag{32}$$

or, in the short hand:

$$Z_{ij}(\tilde{\omega}_j)w_j(a_0) = f_{ij} \tag{33}$$

where, $Z_{ij}(\tilde{\omega}_j)$ = the system impedance which is according to Eq. (2) is,

$$Z_{ij}(\tilde{\omega}_j) = \frac{1}{\psi_j(a_0)\cdot\psi_i(a_0)}\left[\left\{\sum_{k=1}^{n}\left(\omega_k^{\,2} - \tilde{\omega}_j^{\,2} + 2i\,\xi_k\,\omega_k\,\tilde{\omega}_j\right)d_{ki}\,d_{jk}\right\} - \tilde{\omega}_j^{\,2} p_{ji}\right] \tag{34}$$

The complex frequency response $H_{ij}(\tilde{\omega}_j)$ of the system is defined by:

$$H_{ij}(\tilde{\omega}_j) = \frac{\tilde{\omega}_j^{\,2}}{Z_{ij}(\tilde{\omega}_j)} \tag{35}$$

which can be rewritten into a Real-part, $h_{ij}(\tilde{\omega}_j)$, and an Imaginary-part, $\overline{h}_{ij}(\tilde{\omega}_j)$:

$$H_{ij}(\tilde{\omega}_j) = h_{ij}(\tilde{\omega}_j) + i\overline{h}_{ij}(\tilde{\omega}_j) \tag{36}$$

It is known that at the behaviour of complex frequency response $H_{ij}\left(\tilde{\omega}_j\right)$ at resonance case, is in such a way that, its real part, $h_{ij}\left(\tilde{\omega}_j\right)$, vanishes and its imaginary part, $\bar{h}_{ij}\left(\tilde{\omega}_j\right)$, tends to its maximum value.

$$\mathrm{Real}\left\{\frac{1}{\psi_j\left(a_0\right)\cdot\psi_i\left(a_0\right)}\left[\left\{\sum_{k=1}^{n}\left(\omega_k^{\ 2}-\tilde{\omega}_j^{\ 2}+2i\ \xi_k\ \omega_k\ \tilde{\omega}_j\right)d_{ki}\ d_{jk}\right\}-\tilde{\omega}_j^{\ 2}p_{ji}\right]\right\}=0 \quad (37)$$

Eq. (37) provides a first set of $n \times n$ equations for the unknown values of $d_{ik}$ and $\omega_k$. Further, considering the orthogonality condition substituting Eq. (23) and yield:

$$\overline{\Psi}_j^{\mathrm{T}}.\mathrm{M}.\Psi_i=\mathrm{d}_j^{\mathrm{T}}.\phi^{\mathrm{T}}.\mathrm{M}.\phi.\mathrm{d}_i \quad ; \quad \begin{cases}\mathrm{i}=1,2,...,\mathrm{n}\\ \mathrm{j}=1,2,...,\mathrm{n}\end{cases} \qquad (38)$$

where $\overline{\Psi}_j$ is the conjugate vector of $\Psi_j$

$$\overline{\Psi}_j^{\mathrm{T}}.\mathrm{M}.\Psi_i=\mathrm{Real}\,(\mathrm{d}_j^{\mathrm{T}}.\mathrm{d}_j)+\mathrm{Imaginary}\,(\mathrm{d}_j^{\mathrm{T}}.\mathrm{d}_i) \qquad (39)$$

The left hand side of Eq. (30) is real

$$\overline{\Psi}_j^{\mathrm{T}}.\mathrm{M}.\Psi_i=\mathrm{Real}(\mathrm{d}_j^{\mathrm{T}}.\mathrm{d}_i) \quad ; \quad \begin{cases}\mathrm{i}=1,2,...,\mathrm{n}\\ \mathrm{j}=1,2,...,\mathrm{n}\end{cases} \qquad (40)$$

and

$$\mathrm{Imaginary}\ (\mathrm{d}_j^{\mathrm{T}}.\mathrm{d}_i)=\mathbf{0} \quad ; \quad \begin{cases}\mathrm{i}=1,2,...,\mathrm{n}\\ \mathrm{j}=1,2,...,\mathrm{n}\end{cases} \qquad (41)$$

There are $2\times\left[\frac{\mathrm{n}\times\mathrm{n}-\mathrm{n}}{2}+\mathrm{n}\right]=\mathrm{n}\times\mathrm{n}+\mathrm{n}$ independent equations in Eqs. (40) and (41). According to these independent equations and $2n\times n+n$ unknown values of $d_{jk}^{\mathrm{Re}}$, $d_{jk}^{\mathrm{Im}}$ and $\omega_j$ can be calculated by utilizing numerical solution strategy, such as Netwton' s iteration algorithm.

Furthermore, $\Phi_j(x)$ can be obtained by solving the linear equations with known complex coefficient s according to Eq.(23).

## 4. CONCLUDING REMARKS

An efficient strategy is proposed to remove the effect of hydrodynamic pressure in dam-reservoir systems to calculate the natural modal properties of the dam using the data obtained from vibration tests on dams at some water level. As a first step, the simple interactive vibrations of the simple system plate-linear compressible fluid in a finite reservoir are analyzed to verify the solution strategy. The calculated natural frequencies and modes (derived by solving an inverse problem) are compared with the analytical results available for SSSF-plate. Good agreement is observed.

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## REFERENCES

1. Chakrabarti, P. and Chopra, A K., "Earthquake analysis of gravity dams including hydrodynamic interaction". *Earthquake Engineering and Structural Dynamics*, **2**(1973) 143-160.
2. Dominguez, J. and MEDINA, F., "Boundary elements for the analysis of the seismic response of dams including dam-water-foundation interaction effects" II, *Earthquake Engineering and Structural Dynamics*, **6**(1989) 158-163.
3. Höllinger, F., "Time-harmonic and nonstationary stochastic vibrations of arch dam-reservoir-systems", *Acta Mechanica*, **49**(1983) 153-167.
4. Höllinger, F., "Earthquake excited vibrations of elastic dam-fluid systems" (in German). Doctoral Thesis, Technical University of Vienna, 1982.
5. Jablonski, A.M. and Hummar, J.L., "Three-dimensional boundary element reservoir model for seismic analysis of arch and gravity dam". *Earthquake Engineering and Structural Dynamics,* **19**(1990) 359-376.
6. Lei, Y. and Nasserzare, J. :"Calculation of natural frequencies and modes of dams as an inverse problem". Proceeding of the 10th European Conference on Earthquake Engineering, Vienna. Rotterdam: A.A. Balkema., III, (1994) 2107-2112.
7. Parultre, P., Proulx, J., Duran, Z.H., Phat, T.M. and IM, O., "Dynamic testing of outardes 3 gravity dam". Proceeding of the 10th World Conference on Earthquake Engineering, Madrid, Rotterdam: A.A. Balkema, VI, (1992) 3571-3576.
8. Portugal, A. and Caetano, E.S., "Experimental evaluation of the dynamic characteristics of Portuguese dams". Proceeding of the 10th World Conference on Earthquake Engineering, Madrid, Rotterdam: A.A. Balkema, VI, (1992) 3579-3584.
9. Riezinger, J., "Identifikation des dynamischen Verhaltens einer Talsperre bei unterschiedlichen Wasserständen im Speicher" (in German). Doctoral Thesis, Technical University of Vienna, 1996.
10. Riezinger, J., Ziegler, F. and Lei, Y., "Identification problems of large arch-dams". Proceeding of the 10th European Conference on Earthquake Engineering, Vienna.

Rotterdam: A.A. Balkema., III, (1994) 1945-1950.

11. Ziegler, F., Höllinger, F. and Zhang, B., "Random vibrations of dams and offshore structures - A nonstationary spectral approach". In: Stochastic Structural Mechanics, Boca Raton, Fl. (Y.K. Lin, G.I. Schueller, eds.), Berlin: Springer-Verlag, 1987.
12. Nasserjare, J., Lei, Y. and Eskandari-Shiri, S., Computation of natural frequencies and mode shapes of arch dams as an inverse problem, *Advance in Engineering Software*, **31**(2000) 827-836.